

# Computation of cost and effectiveness in decision trees with embedded decision nodes

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## Abstract

**Purpose:** To develop a method of cost-effectiveness analysis (CEA) that avoids the typical restriction that there is only one decision to be made or that the decision is the root node in the tree.

**Methods:** We have developed an algorithm for computing the cost and the effectiveness in decision trees with embedded decision nodes. The method is similar to the traditional evaluation of decision trees, but instead of returning a single utility value for each branch, it returns a list of lambda intervals, each having a cost and an effectiveness. The limits of those intervals are obtained from the incremental cost-effectiveness ratios (ICERs) stemming from posterior decisions.

**Results:** We have built a decision support software tool for performing CEA in decision trees with embedded nodes. It displays the results of the analysis in the form of text or in the form of a cost-effectiveness plot for each decision node and each lambda interval.

**Conclusions:** The method and the software tool that we have developed will permit to perform CEAs for many problems that could not be solved with traditional methods and tools.

## The role of $\lambda$

- $\lambda$  determines the monetary equivalence of health

$$NHB = \lambda \times E - C$$

- $NHB$  = Net health benefit
- $E$  = effectiveness
- $C$  = monetary cost

- $\lambda$  is usually measured in “\$ / QALY” or “€ / QALY”
- $\lambda$  is different for different people, different countries...
- When performing cost-effectiveness analyses,  
 $\lambda$  is unknown

[If  $\lambda$  were known, cost-benefit analysis (one-dimensional) would be more appropriate and easier than CEA (two-dimensional)]

## Role of $\lambda$ in incremental CEA

- Problem: compare two mutually-exclusive interventions

$$NHB_1 = \lambda \times E_1 - C_1$$

$$NHB_2 = \lambda \times E_2 - C_2$$

$$NHB_1 > NHB_2 \Leftrightarrow \frac{C_2 - C_1}{E_2 - E_1} < \lambda$$

- Definition: Incremental cost-effectiveness ratio (ICER)

$$ICER_{2,1} = \frac{C_2 - C_1}{E_2 - E_1}$$

- Conclusion

$$NHB_2 > NHB_1 \Leftrightarrow ICER_{2,1} < \lambda$$

- $\lambda$  determines which option is more beneficial

## The problem of multiple decisions

- The cost-effectiveness (CE) of a decision often depends on subsequent decisions
  - E.g., CE of a test depends on decisions about treatment
- One “solution”: assume there is only one decision
  - Drawback: Many interesting problems in medicine do not satisfy this condition. Unrealistic.
- Other solution: as many branches as possible strategies
  - Drawback: may lead to very large trees, which are difficult to build and debug
- Other solution (TreeAge Pro): assume  $\lambda$  is known
  - Drawback: CEA only makes sense when  $\lambda$  is unknown.

## Our method

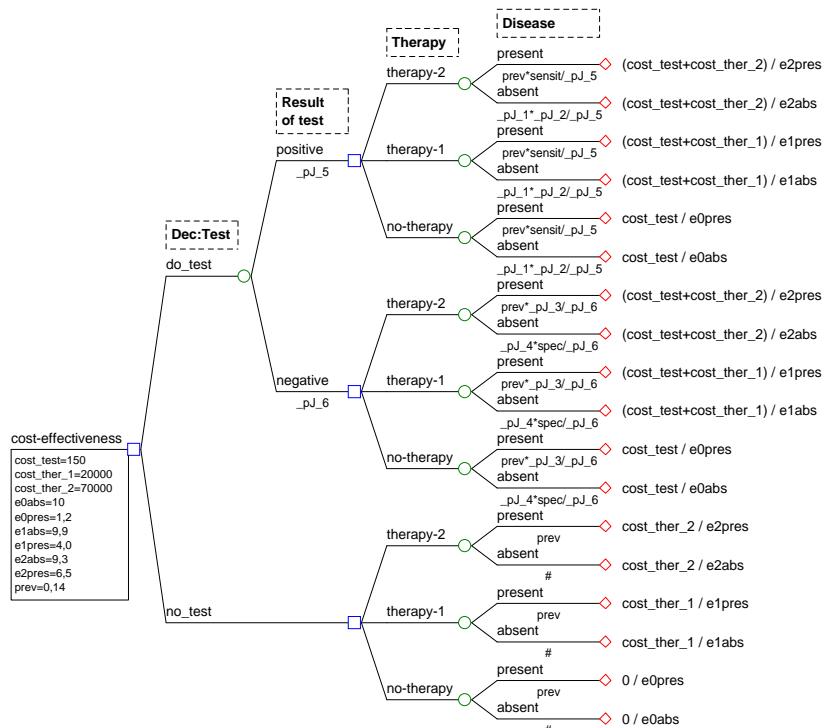
- Build a decision tree with several decisions
- Since  $\lambda$  is unknown, the analysis must take into account all the possible values of  $\lambda$
- $\lambda$  can take on infinite values  $\Rightarrow$  use intervals
- Intervals are determined dynamically when evaluating the tree, and propagated backwards
- Interval partitions from different branches of a node combine at that node
- The solution is given in the form of a partition of intervals

## Example. Statement of the problem

- Disease Prevalence = 0,14
- Test Sens = 0,90 Spec = 0,93  
Cost = 150 €
- Therapy 1 Cost = 20,000 €
- Therapy 2 Cost = 70,000 €
- Effectiveness (QALYs)

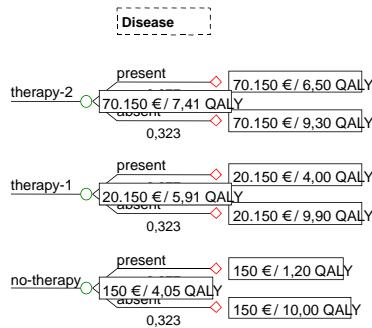
	No therapy	Therapy 1	Therapy 2
Disease present	1,2	4,0	6,5
Disease absent	10	9,9	9,3

- What is the cost-effectiveness of the test?
- What is the optimal policy?



## Example (1)

- For each of the nine “Disease” nodes, take the weighted average of the effectiveness of its branches

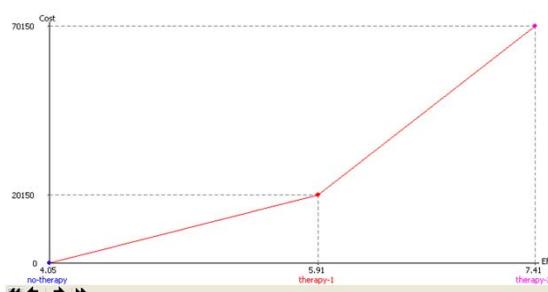


$$eff(therapy) = \sum_{disease} P(disease | result-test) \times eff(disease, therapy)$$

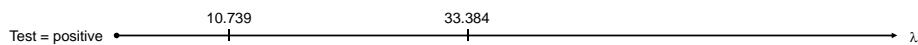
$$cost(therapy) = \sum_{disease} P(disease | result-test) \times cost(disease, therapy)$$

## Example (2a)

- CEA for node “Test=positive”



Interval for $\lambda$	Cost	Effect.	Best therapy
(0, 10.739)	150	4,05	no-therapy
(10.739, 33.384)	20.150	5,91	therapy-1
(33.384, $\infty$ )	70.150	7,41	therapy-2



## Example (2b)

- CEA for node “Test=negative”

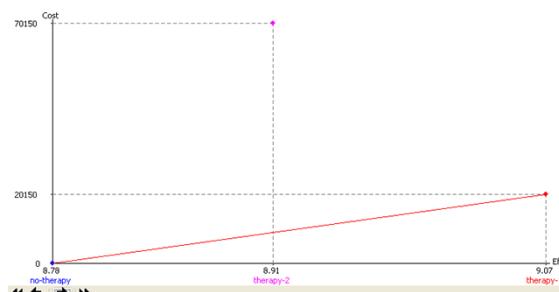


Interval for $\lambda$	Cost	Effect.	Best therapy
(0, $\infty$ )	150	9,85	no-therapy

Test = negative  $\bullet$   $\longrightarrow \lambda$

## Example (2c)

- CEA for node “Dec:Test = no\_test”



Interval for $\lambda$	Cost	Effect.	Best therapy
(0, 65.359)	0	8,77	no-therapy
(65.359, $\infty$ )	20.000	9,07	therapy-1

Dec:Test = no\_test  $\bullet$   $\longrightarrow \lambda$  65.359

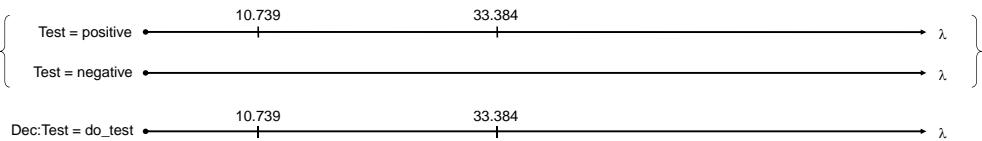
## Example (3)

- At node “Dec:Test=do\_test”, compute the weighted average of its branches **for each  $\lambda$ -interval**

$$\text{eff}(do\_test) = P(\text{pos}) \times \text{eff}(\text{pos}) + P(\text{neg}) \times \text{eff}(\text{neg})$$

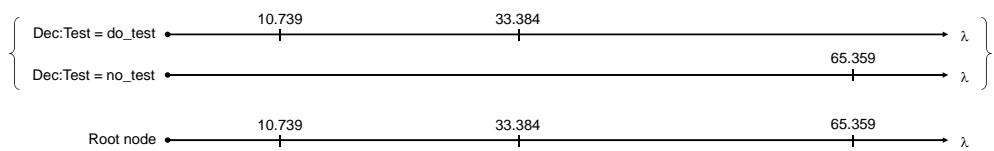
$$\text{cost}(do\_test) = P(\text{pos}) \times \text{cost}(\text{pos}) + P(\text{neg}) \times \text{cost}(\text{neg})$$

Interval for $\lambda$	Cost	Effect.	Best therapy
(0, 10.739)	150	8,77	no-therapy
(10.739, 33.384)	20.150	9,11	therapy-1
(33.384, $\infty$ )	70.150	9,39	therapy-2



## Example (4)

- At the root node (decision about the test), perform a CEA **for each  $\lambda$ -interval**



- Four intervals
  - (0, 10,739)
  - (10,739, 33,384)
  - (33,384, 65,359)
  - (65,359,  $\infty$ )
- Two options: do\_test, no\_test

## Example (4a)

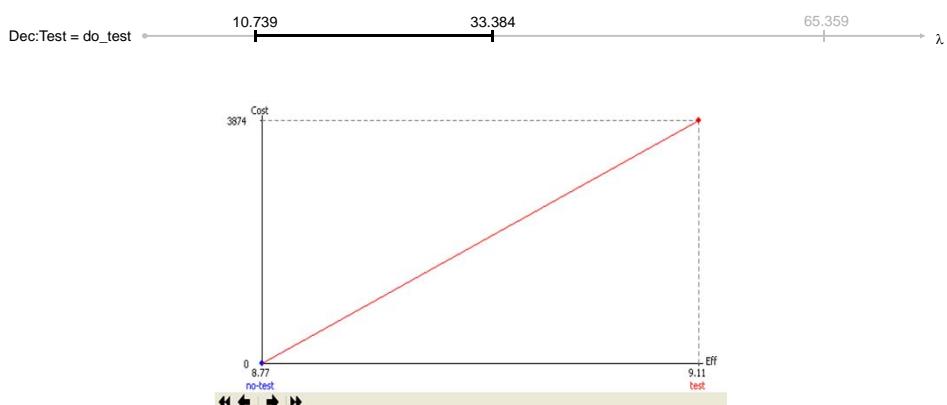
- 1<sup>st</sup> interval: (0, 10.739)



- Option "do\_test" dominated by "no\_test"  
(the same effectiveness, different cost)

## Example (4b)

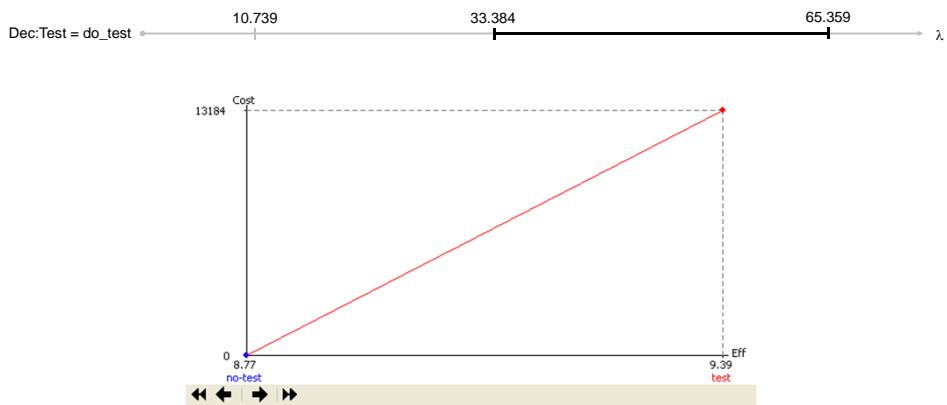
- 2<sup>nd</sup> interval: (10.739, 33.384)



- Threshold at  $\lambda = 11.171$  €/QALY
- This interval is split: (10.739, 11.171) — (11.171, 33.384)

## Example (4c)

- 3<sup>rd</sup> interval: (33.384, 65.359)



- Threshold at  $\lambda = 21.072 \text{ €/QALY}$  (outside this interval)
- In this interval, “do\_test” is more beneficial than “no\_test”

## Example (4d)

- 4<sup>th</sup> interval: (65.359,  $\infty$ )



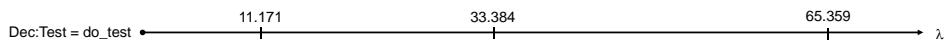
- Option “no\_test” dominated by “do\_test” (the test saves money)

## Example (4e)

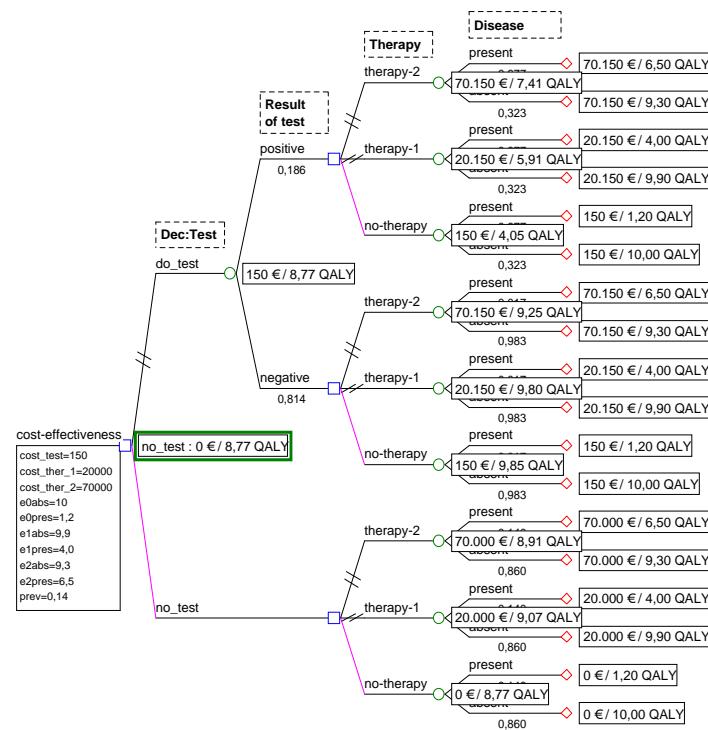
- Cost-effectiveness of the test

Interval for $\lambda$	Cost	Effect.	Best therapy	
(0, 10.739)	0	8,77	no_test	}
(10.739, 11.171)	0	8,77	no_test	
(11.171, 33.384)	3.874	9,11	do_test	}
(33.384, 65.359)	13.184	9,39	do_test	
(65.359, $\infty$ )	13.184	9,39	do_test	

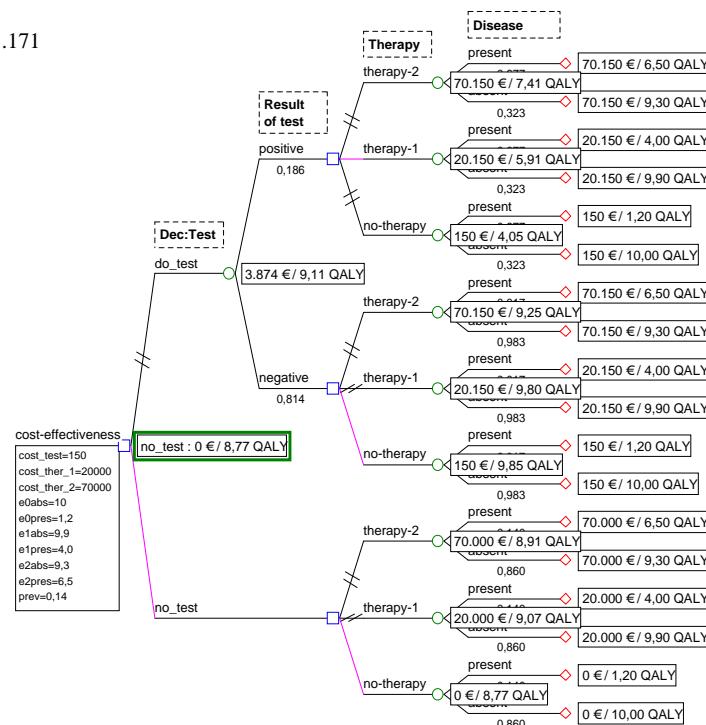
➤ Some intervals merge: only three intervals for this decision



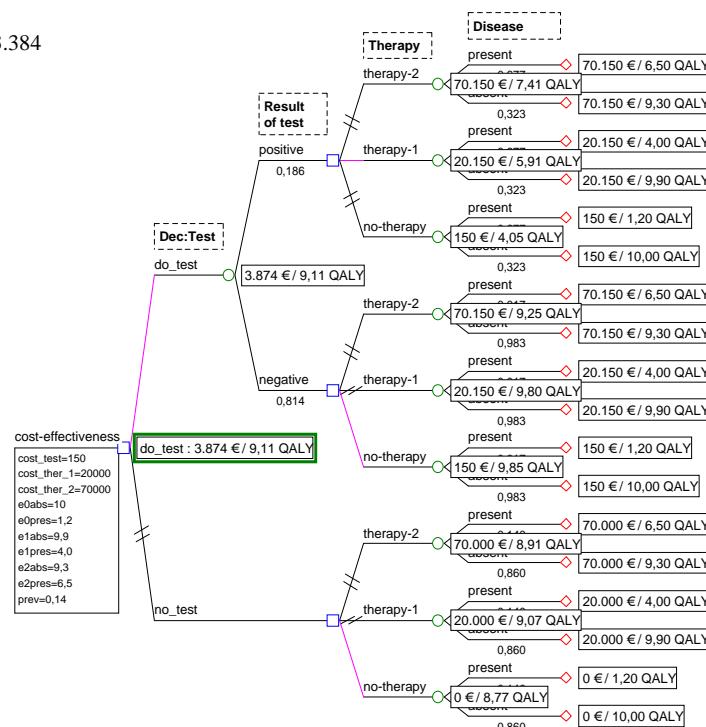
$\lambda < 10.739$



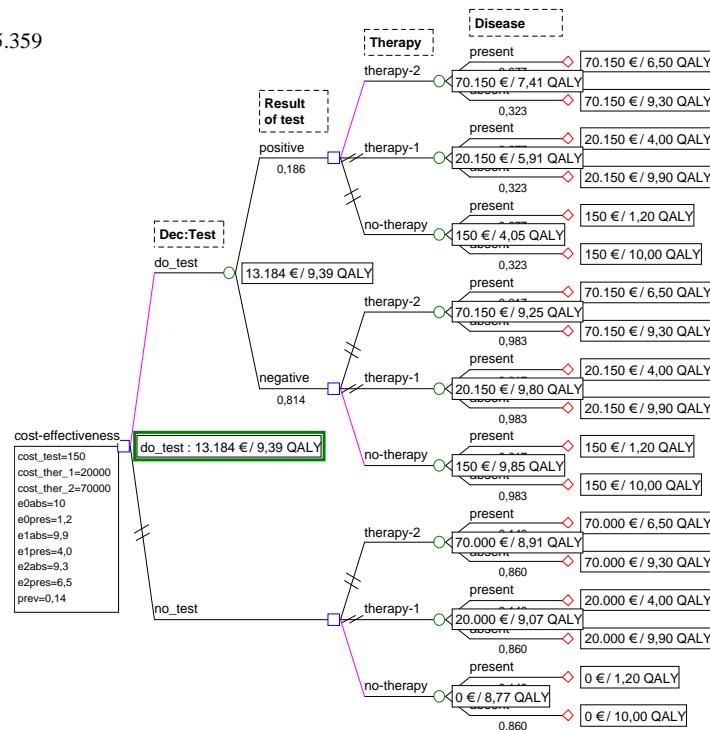
$10.739 < \lambda < 11.171$



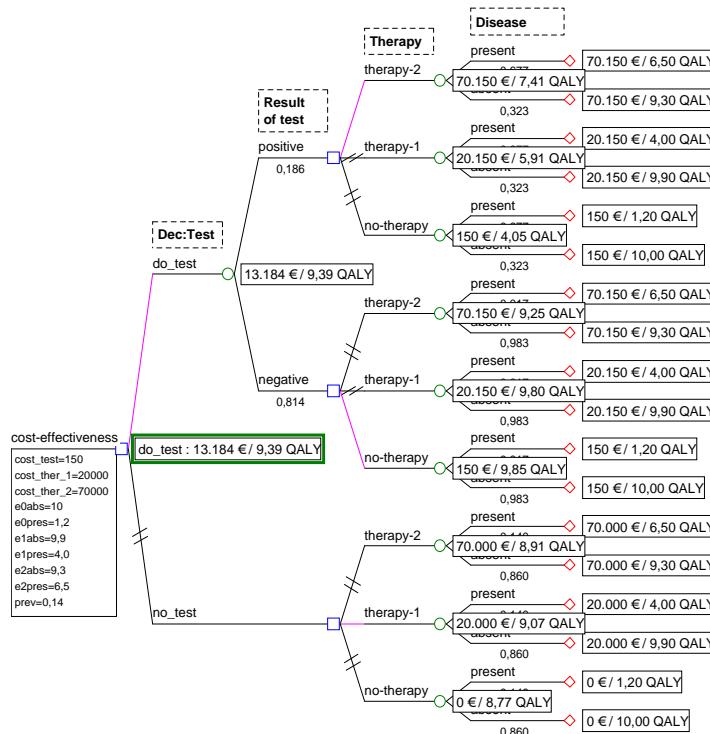
$11.171 < \lambda < 33.384$



$33.384 < \lambda < 65.359$



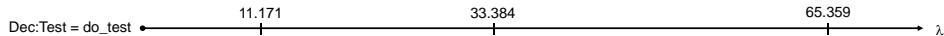
$\lambda > 65.359$



## Example. Conclusion

- Optimal policy

Interval for $\lambda$	Dec:Test	Therapy
(0, 11.171)	no_test	no-therapy
(11.171, 33.384)	do_test	test:positive → therapy-1 test: negative → no-therapy
(33.384, $\infty$ )	do_test	test:positive → therapy-2 test: negative → no-therapy



## Algorithm

- A set of  $\lambda$ -intervals for each node
  - Partition is given by a set of thresholds (values of  $\lambda$ )
  - Each interval has a cost and an effectiveness
  - The intervals and their values are obtained as follows:
- Evaluation of a chance node
  - Partition: union of all the thresholds of its branches
  - The cost and effectiveness of an interval is the weighted average of the values of its branches in that interval
- Evaluation of a decision node
  - Initial partition: union of all the thresholds of its branches
  - Perform a CEA for each interval
  - This may lead to new thresholds, i.e., splitting intervals
  - Two consecutive intervals having the same cost and effectiveness must be merged